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REVIEWS OF BOOKS.

Encyclopedia of the Philosophical Sciences, Vol. I. Logic. By ARNOLD RUGE, WILHELM WINDELBAND, JOSIAH ROYCE, LOUIS COUTURAT, BENEDETTO CROCE, FREDERIGO ENRIQUES and NICOLAJ LOSSIKIJ. Translated by B. ETHEL MEYER. New York, The Macmillan Company, 1913.—Pp. x, 269.[1]

The present Encyclopedia of the Philosophical Sciences was inspired by Hegel's work. It is an attempt at coöperation amongst philosophers without a surrender of individual independence. But if one of the editors, Arnold Ruge, who writes about the underlying purpose of the Encyclopedia, in the introductory paper, saw primarily the idea of *unity* in the various ways of treating logic, the reviewer is impressed with the *multiplicity* of the points of departure.

The discrepancy which is so apparent as regards theories and results is in good part due to the fact that the writers treat of wholly different problems. In the following pages I shall content myself with presenting in some detail the principal results of these various ways of viewing and solving the problem of logic. In a separate paper I hope to sketch these currents of logical theory in their proper setting, examine their mutual relations and disentangle the problems which they attempt to solve.

Wilhelm Windelband contributes the first systematic paper, entitled *The Principles of Logic*. He conceives the task of Logic in so comprehensive a way as to regard it as the "Philosophical Doctrine of Knowledge," the "Theory of Theoretical Reason" (p. 8). He adopts the transcendental method of the Critical Philosophy of Kant and is, as far as his general position is concerned, in agreement with the Marburg School, though the latter is nowhere mentioned in his paper. By taking the fundamental problem so broadly he hopes to unite the many different methods of treating logic in a comprehensive whole and to develop them organically in their articulated order.

The transcendental method of Kant takes the various sciences as a fact and enquires, What are the conditions which make these sciences possible? But Windelband takes as given, besides the sciences, "a mass of immediate experiences of the prescientific consciousness" (p. 9); and the problem of philosophy is "to discover how far general postulates of reason [*Vernunftinhalte*], which are independent of the specific conditions of humanity and which find their justification entirely in themselves, have attained to consciousness and effective value" (p. 9). The Marburg School emphasizes the *objective* aspect of the transcendental method, the reference to the sciences; Windelband dwells on the *subjective* aspect, the reference to human reason. And as there are, for him, three "fundamental psychical activities," namely of *knowing*, *willing*, and *feeling*, there are correspondingly three forms which human culture has taken: *Science*, *Morality*, and *Art;* and three corresponding fundamental philosophical sciences: *Logic*, *Ethics*, and *Aesthetics*.

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[1] The review of this significant volume has unavoidably been much delayed in publication. Since it was written, two of the contributors, Professors Windelband and Royce, have died. I have, however, not thought it wise to reformulate or expunge my comments on their papers.

The first part of the paper entitled "Phenomenology of Knowledge," treats of the empirical material which Logic takes as given. This part is introductory; it does not furnish any *principles* of Logic. Logic itself is divided into "Formal or Pure Logic," "Methodology," and "Theory of Knowledge," which are treated respectively in the remaining three sections of the paper.

In Phenomenology of Knowledge the "fundamental fact" is the distinction "from the point of view of value" (p. 11) between the *true* and the *false*. According to the "immanent theory of truth," as distinguished from the "transcendent" or "representative theory" [*Abbildtheorie*], truth is concerned with the *relation* between ideas, *i. e.*, with judgments. Windelband points out the distinction between the *content* of the judgment and the *attitude* taken towards its truth value, *i. e.*, "the affirmation or negation, the acceptance or rejection, adoption or refusal of the content of the judgment (p. 14)." Whitehead and Russell have recently (in the *Principia Mathematica*) emphasized the logical importance of this distinction, and (following Frege) use a special "assertion symbol" to single out assertions from propositions such as definitions which are not asserted. Windelband seems inclined to doubt, and rightly according to my mind, that this distinction is valid for logic. But his statements are somewhat ambiguous. Does he, or does he not believe that the "moment of assent" is a *logical* aspect of judgment? From his discussion on p. 15, one is tempted to say that he does not consider it so; yet, on the next page, he defines judgment logically as "the assertion of a relation." The difficulty which he urges seems to me to rest on a simple confusion which characterizes his whole position. I may (and often do) assent to a false as well as to a true proposition. My assenting or not assenting is determined by conditions which do not *make* propositions true or false; they belong to a wholly different order of problems. But Windelband makes the truth of a proposition *depend* on this feeling of validity (formal truth). On the contrary, one should say: *Logically* it is a sheer accident that the feeling of validity takes place (at least ordinarily) when we meet *true* propositions. That, as a matter of fact, they often do happen to coincide, must be explained *biologically;* it is not a legitimate problem of logic at all.

The task of Formal or Pure Logic is "to isolate by abstraction those forms of thought which are indispensable to the attainment of truth in perceiving [*Erkennen*] and knowing, and to exhibit their immediate evidence" (p. 22). Formal Logic does not treat of all forms of thought, but merely of the "*forms of right thinking*" (p. 23). This Windelband deems sufficient "to mark out the difference in principle between Logic and Psychology" (p. 23). As the science of the Forms of Right Thinking, Logic is a *normative* discipline. Yet this is only one side of Logic, namely "the side which it turns toward empirical thought" (p. 24). "And here we come on a double aspect of all logical laws: on the one hand they are rules for the empirical consciousness, according to which all thinking which has truth for its aim should be carried on; on the other they have their own inner and independent significance and being, quite independent of the actual happening of ideational processes, which are or are not in accordance wth them. We may call the latter their *value-in-themselves*, the former their *value-for-us*" (p. 24). This distinction seems to me fundamental and of far-reaching importance; but I feel that if it had been carried through consistently in the present paper, it would have led to a very different system of Logic. For does not this 'double aspect' of all logical laws lead to two distinct sciences, the one dealing with the "*objective* meaning" (p. 25), their "inner and independent significance and being" (p. 24), the other with the *subjective* aspect, their coercion for our empirical thinking, compelling of assent in ourselves and in our opponent? The latter, it would appear, even from Windelband's point of view, ought to be a chapter in Psychology; the former Logic properly speaking. Windelband recognizes that the normative constraint of our rational consciousness must be based on *objective* grounds of the thought-content itself; there is "such an immanent inter-connexion that if something is valid [*gilt*], then something else is valid and something else is not valid" (p. 25). Here was a chance for a clear separation of logical from psychological problems, which Windelband in his paper attempts to make. But he has not taken the chance. For him this objective meaning contains for Formal Logic "a presupposition which cannot be proved because it is itself the principle of all proof. The same holds good for the methodological treatment of all the rules of right thinking which Formal Logic can lay down. . . . But if any man stumbles at the fact that when we want to think about thought we must, in so doing, already follow the norms of right thinking—there is no arguing with him" (p. 25). Of course not, because you yourself blocked the way to a valid reply. For can there really be any doubt that you must use your 'laws of right thinking' when you want to think rightly about these laws of right thinking? You are turning in a circle—and blame your opponent for it! Yet, how easy it would have been to escape the dilemma!

The two most pregnant points of view in the doctrine of judgments "are quality and relation" (p. 29). The quality of judgment leads to the laws of thought; and relation between judgments is the subject of the doctrine of categories. This forms the climax of all logical theory (p. 33).

We need not examine the deduction of the system of categories from the principle of synthesis. The whole discussion lacks that rigor which we can to-day demand of similar work. The 'deduction' itself seems to be pure fancy and establishes nothing. There is not even an attempt to prove the necessity and sufficiency of the various categories; and without these a mere list of so-called 'categories' is of no avail. The breadth of the undertaking makes accuracy well-nigh impossible. The whole recent work in Mathematics and Mathematical Logic is completely ignored, even though it represents the most tremendous advance in logical theory since the time of Plato and Aristotle. Windelband has heard that there is such a thing as "Mathematical Logic." But it seems to him a comparatively insignificant affair, if not a vain endeavor: "There are logical principles of Mathematics, but no mathematical principles of Logic" (p. 39). But what does he mean? Is not "Mathematical Logic" in its most important form to-day "Principles of Mathematics"? Or does he mean to deny the possibility of a "Logical Calculus"?

On questions of the History of Philosophy Windelband speaks as a master; on Mathematics and Mathematical Logic not even as an apprentice. This is not an incidental objection—it is vital to his whole method of procedure. Logic, in Windelband's view, presupposes all sciences for its material, and, we may infer, on the part of a contributor to logical theory, a knowledge of these sciences, not superficial, incidental, but intimate knowledge. This knowledge Windelband clearly lacks; and his theory suffers therefrom.

"Methodology" is treated in the third section. First the "*methods of proof and of refutation*" (p. 43) are discussed. This part is not only too brief, but, for the reasons above stated, altogether unsatisfactory. The second part, a discussion of the "*methods of investigation*," contains inspired passages; the living breath of the second edition of the *Critique of Pure Reason* permeates it; and such expressions as "the object's immanent conformity to law" (p. 46), "that *coercion of objectivity*" (p. 47), "the *Logic of the object*," seem to me admirable. But it is a pity that this whole discussion should be so largely occupied with Mathematics and Natural Science, rather than with the Historical Sciences. I, for one, should have been glad to hear from such an expert in this field as Windelband more about the "very fine and thoughtfully developed rules and instruments which, in some cases, stand in rich variety at the disposal of the *criticism* and *interpretation* of tradition" (p. 52), about "philologico-historical hermeneutics" (p. 53) and the rôle of the "logic of hypothesis" (*ibid.*) in this realm. They are referred to, but not discussed.

'Theory of Knowledge' inquires into "the relation of knowing to reality" (p. 55), *i. e.*, "the relation between that which is objectively thought and that which exists" (pp. 55–6). The central difficulty is met when we determine the nature of the 'something' which even Pure Mathematics presupposes and "whose existence and essence is quite independent of whether or not a knowing consciousness takes it for its object" (p. 57). To ascribe to it a 'Being' peculiar to itself has always led to Mysticism, according to Windelband. He, instead, follows the lead of Lotze who coined the term 'validity' for this peculiar 'something' which has "neither substantial nor functional reality or actuality" (p. 51). By 'validity' Windelband means "nothing else than the sum-total of the *inter-connexions* and *relations* between existents. They are not themselves existents, either as things, as states, or as activities; they can only become 'actual' as the content of psychical functions of knowing. But in itself the *realm of the valid* is nothing else than the *form and order under which that which exists is determined*" (p. 59).

It is quite obvious that under the title of Theory of Knowledge Windelband includes all that which is usually called Metaphysics; and he himself speaks of it as "Critical Metaphysics." It is of course impossible in a review to enter into a discussion of so broad a question, particularly if the theory itself is stated so briefly. I can only say, somewhat abruptly, that I do not feel as hopeful about it as the author. From the mathematical point of view it seems quite insufficient to limit the 'valid,' as expressive of the peculiar 'something' which mathematics presupposes, to the *relations between* that which exists. And the mathematical relations are, strictly speaking, not at all between that which exists, but between 'somethings' *themselves mathematical*, call them what you please, *i. e.*, points, lines, number, collections, etc.

It would seem simpler to *begin* by assuming a special kind of existence, namely *mathematical* existence, and to introduce the general idea of *existence appropriate to a particular Universe of Discourse.* This allows us to introduce as many different kinds of existence as we need to get a particular science started. *Afterwards* we may raise the question of the precise relation between the different kinds of existence, *e. g.*, mathematical existence and physical existence. This is, of course, what a mathematician of to-day actually does. He is thereby enabled to begin without being obliged first of all, and before he has moved a step in mathematics, to decide on the most fundamental questions of metaphysics. And it is just as important for the metaphysician to proceed thus, for otherwise he cannot even state his problem.

The main purpose of Royce's paper on "The Principles of Logic" is to give a brief exposition of the doctrine which he calls the "*Science of Order*"; of it "traditional General, or Formal Logic is but a part, and in fact a very subordinate part" (p. 67).

The first section gives a general review of Methodology. It indicates summarily "how Logic as Methodology differs from and yet gives birth to Logic conceived as the Science of Order" (p. 69). This is shown for the *Deductive Method;* and similarly for the *Inductive Methods*, of which Royce mentions the *Method of Classification* (p. 74), the *Comparative Methods* (p. 75), the *Statistical Method* proper (*ibid.*), and especially what he calls the *Organized Combination of Theory and Experience* (p. 78).

Royce calls attention to the contribution made to the theory of induction by the American philosopher Charles S. Peirce.[1] The usual presuppositions, on which "our right to generalize from a limited set of data" (p. 79) are founded, namely the *uniformity of nature*, and the *principle of sufficient reason*, are not sufficient; nor are they always necessary. But to make generalization possible it is sufficient to assume that the "facts, and so the whole aggregate of them, whatever they are, have *some definite constitution*" (p. 83). Assuming such a definite constitution, we can find out what this constitution is by the "*method of sampling*," provided the samples are "fair samples." A "fair sample" of a collection "is a sample concerning which we have *no reason to suppose that it has been chosen otherwise than 'at random,'* or in a representative way" (p. 87). This method of sampling becomes particularly important in its application to the testing of hypotheses.

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[1] This seems an opportune occasion to call the attention of such organizations as the American Philosophical Association to the desirability of having the philosophical writings of Charles S. Peirce collected and published in a suitable form.

But the fact which for Royce stands out most prominently in the present connection is this: The "Organized Combination of Theory and Experience," this "most perfect of the scientific methods" (p. 91), requires "concepts and systems of concepts which admit of precise and extended deductive reasoning" (*ibid.*).

A review of Methodology leads thus to a discussion of the problems of the Science of Order, and in the second section Royce gives a general survey of the "*Types of Order*." This section is a statement, in Roycian terms, of recent investigations by mathematicians and mathematical logicians into the properties of relations, of classes, and of propositions which are the basis for the definition of "types of order" such as are used in the sciences. The most significant of these is "Serial Order." Instances of it are Numbers and Quantities. The application of these principles to the "theory and description of more complicated order-systems" involves the "Correlation of Series" upon which the whole theory of Mathematical Function rests (p. 116). The most important problem, from the point of view of deduction, is the problem of *elimination* of which the traditional "theory of the syllogism" is but a very special application.

This whole section calls for little critical comment; I mention two points only; the first concerns the relation of Classes and Relations, the second the concept of Classes itself.

1. "Relations are impossible unless there are also classes" (p. 106). There is no question that "relations" and "classes" are closely connected. But it is not warranted to assume, as this passage does, that relations *presuppose* classes, nor is there a *necessary logical* connection between the two. At all events Whitehead and Russell have made, in the *Principia Mathematica*, what seems to me a successful attempt to develop a theory of relations based on propositional functions involving merely *individuals;* classes occur as symbolic expressions, as

"quasi-things," but their existence is not assumed and they are in no way needed to define relations. Of course I do not mean to deny by this that relations *may* be defined as "Classes of Couples" (Peirce, Schroeder).

2. "The concept of a Class, in the logical sense, depends (1) Upon the concept of an . . . *Individual* . . . ; (2) Upon the concept of the *relation of belonging to* . . . ; (3) Upon the concept of *assertions* . . . ; (4) Upon the concept of a Principle . . ." (p. 106). It is undoubtedly true that an analysis of what we mean by a 'Class' will furnish these ingredients; but it is to be questioned whether this justifies the statement that the concept of Classes logically *depends* on them. To be specific, it is perfectly possible to develop a theory of classes without introducing the concept of an individual; in fact E. Schroeder has done so in his *Algebra der Logik* and this is one of the cardinal features of his work, as he defines what he calls an Individual on the basis of his Calculus of Classes.

It is to be noted, however, that in this presentation of logical doctrine the "*will to be rational*" (p. 111) already makes its appearance. The concept of an individual (and the same is true for that of classes) involves for Royce an "attitude of will." But these attitudes are not wholly arbitrary or of merely relative value: "We have an absolute logical need to conceive of individual objects as the elements of our ideal order-systems" (p. 107). This is further developed in the third section, which sketches "the logical genesis of the types of order"; it is the most original part of the paper. All the rest may be considered as introductory. Here he develops and justifies what he terms "Absolute Pragmatism." The "*central problem*" is: "What are the necessary 'logical entities,' and what are their necessary laws? What objects must the logician's world contain? What order-systems must he conceive, not as contingent and arbitrary, but as so implied in the nature of our rational activity that the effort to remove them from our world would inevitably imply their reinstatement, just as the effort to remove relations and classes from the world would involve recognizing both classes and relations as, in some new way, present" (p. 125).

For a solution of this central problem of the Theory of Order, Royce refers to two papers by A. B. Kempe, and his own paper published in 1905, in the *Transactions of the American Mathematical Society.* This paper developed the properties of the system which Royce calls a system Σ. The entities of logic satisfy the laws of the system Σ. The present paper shows that "*modes of action*" are a set of entities which form a system Σ. "The result . . . is that it at present appears to be possible to define, upon the basis of purely logical relations, and upon the basis of the aforegoing principles concerning rational activity, an order-system of entities inclusive not only of objects having the relation of the number system, but also of objects illustrating the geometrical types of order, and thus apparently including all the order-systems upon which, at least at present, the theoretical natural sciences depend for the success of their deductions" (p. 134).

The preceding summary of Royce's masterly paper, has, I hope, at least brought home to the reader the fundamental nature of the problems which he treats, and the originality of the solutions which he gives. The paper is an example of the complete transformation which is taking place to-day in logical theory.

With most of what Royce says I find myself in entire agreement. His whole conception of logic as the Science of Order is bold and admirable. His discussion of the difficulties involved in the concept of an individual brings out points too easily taken for granted in current theories which point to psychological facts in order to solve logical difficulties. I agree with his rejection of self-evidence as a criterion for fundamental principles. Of particular importance is his statement that the principles of logic are neither purely empirical propositions nor purely arbitrary assumptions. If they are *hypotheses*, why are they *necessary?* To treat them *merely* as assumptions, as most mathematicians do, is insufficient. Here is a fundamental problem which calls for a solution, and Royce's solution certainly has much that is persuasive.

By way of criticism the following: Royce's theory of "Absolute Pragmatism" rests on the following grounds: His criterion of "absolute validity," his theory of the "Will to be Rational," and his introduction of "modes of action" as entities which form a system Σ, *i. e.*, satisfy at least all the laws of logic.

1. The criterion which Royce uses to determine '*absolute validity*' does not accomplish what he supposes it does. Of course I do not deny the principle itself, namely that 'a proposition which is implied by its own denial is true.' But this principle is *on a par* with other principles of logic. It does not in any way establish *absolute* validity.

2. Regarding the "rational will" or "the will to be rational" I can see that such an entity, if it existed, would furnish a *point d'appui* for the hypotheses of logic. But I agree with Mr. Russell in thinking that this rational will is irrelevant to the principles of Logic. The

clean separation of the logical problem from the psychological is important and necessary both for logic and for psychology. I prefer therefore a solution of the problem stated above which avoids the danger of complication with psychology. I have attempted such a solution in a paper entitled "The Generating Problem."[1]

3. I must confess to some difficulty in understanding precisely what kinds of entities modes of action are. They are distinguished from actions. Royce illustrates his modes of action by such examples as 'to sing' or 'singing', 'to dance' or 'dancing,'—in general 'to do something' or 'doing something.' This is clear as far as it goes. The difficulty comes when we try to understand what is meant by the 'contradictory' of such a mode of action. In what sense is 'not to sing' the contradictory of 'to sing'? We can readily see how 'I sing' is incompatible with 'I do not sing' or how a *class* of actions such as 'singing' is to be distinguished from the class of actions 'not to sing.' But in the one case we are dealing with propositions; in the other with classes, *neither of which is what Royce means.* Modes of actions are "open to any rational being"; but it seems quite necessary when we speak of contradictory modes of action that the reference be to the same individual or the same rational being, at the same time. These difficulties have their origin in the definition of the 0 mode and the 1 mode of action. The first is illustrated by the phrase 'to do nothing' or 'doing nothing,' the second by 'to do something' or 'doing something.' If we symbolize a mode of action such as 'to sing' by A, the contradictory by $\bar{A}$, then between the two ought to hold the following two relations: $A\bar{A} = 0$, $A + A = 1$. Products and sums have been explained; accordingly we have to translate these symbolic statements as follows: 'To sing and at the same time not to sing is equivalent to doing nothing,' and 'to sing or not to sing is equivalent to doing something'! Again, if A stood for a class of actions or for a proposition, the meaning of these two propositions, $A\bar{A} = 0$, $A + \bar{A} = 1$, would be clear. But in this application to modes of action as distinct from actions, they are not clear to me. However, I do not urge this as an objection to Royce's theory; but here are difficulties.

The first and longest part of Couturat's paper on "The Principles of Logic" comprising paragraphs 1, 2, 3, and 4, gives an outline of the principles of modern logic, or what has been variously called 'Symbolic Logic,' 'Symbol Logic' (Mrs. Ladd Franklin), 'Mathematical Logic,' 'Algebra of Logic,' and what Couturat, in previous writings, has proposed to call 'Logistique.' The second part, paragraph 5, treats of methodology; the third, paragraph 6, of the relations between logic and universal language.

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[1] *Cf. Journal of Philosophy*, etc., Vol. X, No. 3.

The first part, being itself a brief exposition of a large body of doctrine, more or less universally accepted among modern logicians, would call for very little comment, were it not for the fact that a number of rather confusing mistakes have crept into the account. Couturat has made a reputation by his lucid exposition of modern logic, principally, though not exclusively, of the ideas of Mr. Bertrand Russell in *Les Principes des Mathèmatiques.* In the present paper his symbolism "approximates most closely to Schroeder's" (p. 138). But a quotation from Bertrand Russell introduces his paper, and Bertrand Russell is present throughout.

It is appropriate that an account of 'Symbol-Logic' should be criticized first on the ground of the symbols which it uses. Couturat has deviated from Schroeder's symbolism in a number of important points. It would seem a reasonable principle that such deviation is justified only if thereby an improvement can be obtained. This is not the case here. He changed the symbol for inclusion and implication, using the one which he had already used in his brief book on *L'Algebre de la Logique.* This symbol has nothing to recommend it except the comparative ease in writing it. I am holding no brief for Schroeder's symbol; but if it was to be rejected, it would have better been for Charles S. Peirce's symbol, which Royce has adopted, or better still for Mrs. Ladd Franklin's.

As little can be said in defense of Couturat's symbol for negation. It is a modification of Schroeder's earlier symbol for negation, but it is noteworthy that Schroeder himself when, in his third volume, he came to the Algebra of Relatives, reverted to the horizontal line for negation, which seems by all means better than either Schroeder's earlier or Couturat's present symbol.

More serious is the objection to Couturat's symbol for a relative product, namely a star: (*). It is symmetrical, whilst the operation which it symbolizes is not symmetrical. (I call an operation 'symmetrical' if it obeys the commutative law.) Besides, the symbol is inconvenient. I can think of no good reason why Couturat should have abandoned Schroeder's altogether satisfactory symbol.

For the converse of a relation R, Couturat uses the symbol R^c. This is very bad. According to his own usage, this ought to be read: R to the power of ^{c}. Again I do not see why he did not keep Schroeder's well-suited symbol $\check{R}$, which even such iconoclasts as Whitehead and Russell have adopted in their *Principia Mathematica.*

Following Peano (and Russell) Couturat writes the "variables as indices after the copula" (p. 150) to express: "is equal to for all values of x," or "implies for all values of x," etc. How inconvenient this may be, is shown on p. 161, where two formulae are altogether distorted, by printing the ϕ somewhat too high and giving the expression a wholly different and erroneous meaning.

About the force of these objections to Couturat's innovations, so far as symbols are concerned, there may perhaps be a difference of opinion; I do not think there can be any difference of opinion regarding the objections which have to be made to some of the contents of the paper. *Operations* and *relations* are confused with each other on page 141: "Negation is an operation or relation fundamental for logic." Now it is true that operations in logic can be *expressed* in terms of relations, but the two are very different. Negation is an operation, not a relation. The paper confuses concepts (and classes) with propositional functions, just as it confuses relations with propositional functions. "A concept is a propositional function with one variable" (p. 157). Of course Couturat knows that a concept or class *is* not a propositional function, but is *determined* by a propositional function. To use Mr. Russell's symbolism, it is not $\phi(x)$ which represents the class, but $x\phi(x)$. Similarly for relations: "What, then, is a relation? . . . it *is a propositional function having several variables*" (p. 170). "Let the relation be: 'X is the father of Y'" (p. 179). Of course "X is the father of Y" is not a relation but a *propositional function,* to use Russell's expression, or a "*Relativ-Coefficient,*" to use Schroeder's. 'Is the father of' is the relation and a relation is therefore not represented by $\phi(x, y)$, but, *e. g.*, by $xy\phi(x, y)$, according to Mr. Russell's usage.

It is a consequence of this confusion between relation and propositional function that *inclusion* of relations and *implication* of propositions should be confused with each other. "To say that R_1 is contained in R_2 is the same as to say that it *implies* it" (p. 172). The symbolic statement here is correct, but the English equivalent just quoted is incorrect. It ought to read: "To say that R_1 is contained in R_2 is equivalent to saying that the corresponding propositional function (xR_1y) implies the propositional function (xR_2y)."

The definition of relative multiplication, "the most important operation in the calculus of relations" (p. 174), is wrong. It ought to read: "If there are given two relations R_1 and R_2, *and if there exists a y, such that* . . . " the rest remaining as on p. 174. On p. 163 Couturat reads the symbolic statement: $a\bar{b} = 0$ as follows:

"It really expresses the nullity (non-existence) of the class $a\bar{b}$: '*a* which is not *b* does not exist.'" What he ought to say is: "The *a*'s which are not *b*'s do not exist," or "There is no *a* which is not a *b*," *i. e.*, "There is no *individual* or *member* of the class $a\bar{b}$," *i. e.*, "The class *ab* is a null class." But of course the null-class exists (Bertrand Russell to the contrary notwithstanding), *in the same sense* in which any other class exists.

It is to be regretted that Couturat's paper, at least in this first part, is marred by these imperfections. In spite of the great amount of original work done in this field there exists, as far as I know, no other elementary exposition of this modern logic in English. It is to be hoped that these imperfections will be removed in a new edition of the volume.

In the second part of his paper, Couturat treats under the title "Methodology" problems which belong to the Study of the Structure of Systems, and to Critique of Cognition.[1] On the latter point Couturat briefly and admirably summarizes the results of the recent work in this field. His views on the Structure of Systems, however, though supported by good authority, seem to me strangely inconsistent. It is impossible to enter into detail here. I will merely illustrate my point by the following two quotations: "Thus all the scientific and objective contents of a theory are contained within its first notions, and in its axioms or postulates" (p. 186). This is a common enough, though wholly erroneous opinion. Contrast with it the following: "Definition, then, is an essential element of demonstration: for, indeed, how can we hope to establish anything whatever about a term, if not by taking account of its meaning, of its conceptual content? Now this content is furnished by the definition and by it alone; for it is forbidden to attribute to a notion any element which does not figure in its definition, *i. e.*, in its construction" (p. 183). Couturat seems to admit here, and rightly I believe, that definition furnishes *new content* to a deductive system. Yet this is denied, or at least ignored, in the previous quotation. Couturat's fundamental view of the nature of a deductive system is contained in a noteworthy passage on pp. 186–187 which but for lack of space I should quote in full. The idea of "the reciprocal interlacement of all the propositions" here stated seems to me very fundamental. Schroeder[2] referred to it; it forms the basis of Mrs. Ladd Franklin's

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[1] For an explanation of these terms see my papers: "Studies in the Structure of Systems," especially the first: *Journal of Philosophy*, Vol. IX, No. 8.

[2] See *Algebra der Logik*, Vol. II, part 1, p. 55.

Doctrine of Histurgy;[1] and the recent work of mathematicians and logicians only tends to verify and amplify the divination of the earlier investigators.

In late years Couturat was mainly occupied with the fascinating problems of universal language, and as he was led to these problems through his logical work, it is fitting that his present paper should conclude (Chapter VI) with a discussion of the relation between logic and language. Much in the recent work in mathematical logic, prominently by the school of Peano, is guided by ideas similar to those prompting the more popular and utilitarian endeavors of universal language-builders. The logician who is able to express his propositions wholly in his own symbolism, as, *e. g.*, the authors of the *Principia Mathematica*, have almost, though not quite, succeeded in doing, has in so far invented, or used, a universal language, at least for his special field. But it is a language without lingua. How far the ideas set forth by Couturat are acceptable, I am unable to judge. But he pleads the cause of universal language strongly and impressively.

Frederigo Enriques sketches in his paper, "The Problem of Logic," the leading principles of a Theory of Knowledge which may be called *Critical Positivism* (p. 217). "The aim of Critical Positivism . . . is to carry out the programme as laid down in Kant's *Critique* . . . in a scientific spirit consonant with that in which it originated" (p. 239).

The first part treats of Logic as the "Science of Exact Thinking." Out of this very concise chapter I can mention only the following point. "Thought takes for its objects . . . things from the perceptible world . . . . But, in defining or drawing inferences from these things, it always presupposes *certain conditions*,[2] in virtue of which these data are taken as 'objects of thought'" (p. 218). It seems then, if we may so express it, that the conditions *transform* the ordinary *thing* into an *object of thought*. This is, according to Enriques, only one of four ways in which thought posits its object. But it seems to be the most important because it paves the way for the critical solution of the problem of general ideas (p. 234). And the idea of conditions which certain entities must satisfy in order to become objects of thought is fundamental in all ways in which thought posits its objects.

Much more original however than this "Logic as the Science of Exact Thinking" is the second part which treats of "Logic and

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[1] See Mrs. Franklin's paper on Histurgy, read before the American Philosophical Association in Baltimore, 1908. A briefer form of it was published in the *Proceedings of the Heidelberg Congress of Philosophy*, 1908.

[2] Italics mine.

Reality." Its problem is: How is the application of logic possible? This problem is important whatever our theory of logic may be. The author investigates two problems, which are intimately related, namely: "1. . . . whether and how the immutability of logical objects implies the persistence of a corresponding something in the world of Reality. 2. Whether there is anything in Reality corresponding to general and abstract concepts" (p. 231). He presents, as a solution of the first, the Doctrine of Critical Positivism which is summed up in the following proposition: "(*a*) . . . the objects of logical thought are unchangeable. (*b*) . . . Reality itself is changeable, but . . . within the flux, things and relations can be distinguished which change so slowly that they may be treated as logical objects. (*c*) . . . the approximate correspondences established between logical objects and changing realities admit of progressive correction by the help of deduction and experimental verification, and . . . in this way new and stricter immutabilities can be arrived at" (p. 232). This seems to me admirable as far as it goes. The fundamental principles of logic are here considered as conditions, which certain entities, namely, the logical entities, satisfy; for them the properties which are deduced from the fundamental propositions are valid; and any entities which satisfy the fundamental conditions approximately, will also have approximately these further properties. The solution of the second problem is perfectly analogous.

A further problem concerning the relation of 'Logic and Reality' is "to determine how the acquirement and extension of scientific knowledge takes place" (p. 236). It is treated under the title "The Problem of Methodology."

Attention is called to the rôle which hypothesis plays in induction. Enriques distinguishes between *implicit* and *explicit* hypotheses, and concerning these two he notes the following important points: "(*a*) Under artificially simplified conditions, the result of experiment may offer an affirmative or negative answer to the question formulated in the hypothesis, *when the implicit hypotheses have been correctly apprehended.* (*b*) More often, however, such results confirm or refute not a single hypothesis, but an inference which follows from a *system of hypotheses*" (pp. 237–238). "Implicit hypotheses must be regarded as the least variable element in our systems of knowledge. . . . *The confirmation of such implicit hypotheses does not depend on the success or failure of definite experiments, but upon the congruence of experimental results as a whole: the criterion of implicit hypotheses is the Axiom of Contradiction*" (p. 238).

To this it may be said that the tendency of all scientific work is to make the implicit hypotheses explicit as far as possible; that all hypotheses are to be judged, as far as their validity is concerned, by the same criteria, and that the Axiom of Contradiction is only one of these.[1]

I am impressed with the importance of the theories presented here, but the value of the author's work would have been enhanced, if he had not brushed aside too easily mathematical logic. "A new school of 'Logistic' or 'Mathematical Logic' has arisen. Its special object is the revision of the Aristotelian schemata" (p. 217). But the scope of Symbol-Logic, whatever it may have been in the beginning, has far outgrown "the revision of the Aristotelian schemata." Much of the work done by these mathematical logicians could have profitably been used in the present paper.

Benedetto Croce treats in his delightful brief paper with humor, paradox, and sarcasm, of the "Task of Logic." He will have nothing to do with the endeavors of the Logisticians. They are "in sorry case; they are like peddlers who carry around specious and glittering wares, crying them up to the public and advertising their utility for all sorts of purposes, but who fail to attract customers" (p. 200). Croce himself is not a customer. Mercy no! "The austerity of the new formulae frightens us" (*ibid.*).

Neither is he concerned with "those normative and formal logicians who, in prescribing norms, practically prepared the way for the more radical demands of the Logisticians themselves" (p. 201). For Formal Logic, "Descriptive Logic" as he prefers to call it (p. 201), as well as Logistic, "can, at best, but furnish us with convenient summaries, which are as useful for the memory as they are insignificant and uninforming for the understanding" (p. 201); whilst Croce's Logic "is essentially a *doctrine*, a *theory*, a *science* . . . it is a theory, entirely devoted to the task of inquiring into the nature of thought, as exemplified in science as a whole and in the particular sciences" (p. 200). Logic, Philosophical Logic, is a science, it is a true science, it is the only true science (p. 201), it is "the whole of philosophy" (p. 204). For "Logic as a special science is unthinkable and incomprehensible" (p. 203) and philosophy is an "unanalyzable unity" (p. 204). Logic is "the science of true science or of the philosophical concept. . . . It is the philosophy of Philosophy" (p. 209).

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[1] *Cf.* my paper: "Critique of Cognition and Its Principles." *Journal of Philosophy*, Vol. VI, No. 11.

There is something whole-hearted about Croce's manner. He despises the specialist; he does not believe in the separation of problems: "We take up the cudgels here against the *separatists*" (p. 204). I must confess myself to be an ardent separatist and I heartily recommend all opponents to read Croce's essay; it is the best implicit argument *for* the separation of problems that I know. Logic, Philosophy (for the two are in some mysterious way the same) is indeed many queer things in Croce's view; it is not Poetry, as he rather reluctantly argues; but it is Religion. "We have assumed throughout that the identity of Religion and Philosophy has been both exhibited and recognized" (p. 210). It is also History: "Philosophy itself is History" (p. 211). And a certain aspect of philosophy is Metaphysics: "In this second sense Logic has rightly assumed, on occasion, the name of Metaphysic" (p. 215). I have not fully grasped the arguments by which these various propositions are established; nor am I vitally interested in trying to understand them. No doubt they are comforting to some souls.

In the first part of his paper, *The Transformation of The Concept of Consciousness in Modern Epistemology and its Bearing on Logic*, Nicolaj Lossikij develops an interesting concept of consciousness, which in the second part is applied to the problems of logic. "Consciousness is the sum-total of everything which stands in a certain unique Relation to the Ego. This relation is simple, and cannot therefore be described or defined by analysis into its elements. We can only indicate it by the following expression, which is metaphorical only, and must not be taken literally. Everything falls within the sphere of consciousness which the Ego 'has'" (p. 242). "According to this theory every fact in Reality with which *I am acquainted* is not merely a fact, it is also, owing to the circumstance that my Ego is connected with it by the relation of 'having-in-consciousness,' a content of consciousness; in other words, the Ego exercises towards it the *function* of becoming conscious" (*ibid.*).

Of this 'having' two kinds are distinguished, the first of which is designated as 'mine,' and the second as "given-to-me" (p. 247).

This new conception of consciousness entails, according to Lossikij, important modifications within the sphere of logic. The present paper however examines only a few of these. "In modern philosophical literature the doctrine that knowledge implies synthesis as well as analysis is widely held. Analysis is certainly an activity of the individual knowing. But whence comes synthesis?" (p. 249). According to the present theory "synthesis is given in the constitution of the object" (p. 249).

This leads to a new statement of the relation between subject and predicate: " . . . the judgment should be thrown into the form: 'Where *S* is , *P* necessarily is also.' The relation thus abstractly conceived must be called a relation of *functional dependence* . . . , in so far as we are dealing with combinations of elements . . . which lie outside the act of cognition; when, however, we are considering how far this same interconnection enters into the structure of the judgment or of the syllogism, it must be called a relation of *ground and consequent*" (p. 263).

These criticisms of the traditional theory of the relation of subject and predicate seem to me just, and the emphasis on the concept of the relation of functional dependence excellent. It should be extended; it plays too small a rôle in our usual theories of logic. The theory of truth which is based on this distinction between functional dependence on the one hand and ground and consequent on the other, seems to me open to criticism. But there is not room here to state my objections.

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Critical Realism: A Study of the Nature and Conditions of Knowledge. By ROY WOOD SELLARS. Chicago and New York, Rand McNally and Company, n.d.—Pp. x, 283.

The reader who takes up this volume, having his mind filled with the suggestions which current literature has caused to cluster about the word 'realism,' is likely to be disappointed; unless he is a convinced and satisfied realist, he will be happily disappointed. The emphasis of the title lies upon the word 'critical' rather than upon 'realism' and the onus of the criticism falls as much upon recent realism in this country and in England as upon idealism or pragmatism. I do not mean to suggest that the name realism is ill-chosen, for the author no doubt may properly be called a realist, but he is entirely frank in admitting his sympathy with some parts of idealism and pragmatism, and his purpose is avowedly to construct a theory of knowledge which shall do justice to both these positions as well as to realism. Indeed, he regards realism in its current form as having the same vices as idealism and shares with it little more than its most general conclusion, that there is knowledge of a non-mental reality, a conclusion at which he arrives, however, by another road.

"My thesis is, then, that idealism and realism have had essentially the same view of knowledge and that the large measure of sterility